



CARIBBEAN ACTUARIAL ASSOCIATION



Accredited as a Full Member
of the International Actuarial Association

"Crossing the Boundaries of Tradition"

19th Annual CAA Conference

Rose Hall Resort and Spa, a Hilton
Resort in Montego Bay, Jamaica.

December 3-4, 2009



CARIBBEAN ACTUARIAL ASSOCIATION



Accredited as a Full Member
of the International Actuarial Association

Opportunities Beyond the Frontier

The Promise of Stochastic Approaches

Presented by
Roger M. Hayne, PhD, FCAS, MAAA

Here There Be Dragons

A Map of Our Journey Today

- Comfortable lands where we dwell
- Some reports from the frontier
- Sailing past the edge of the world
- Redrawing the map

COMFORTABLE LANDS WHERE WE DWELL



Traditional Forecast Methods

- Forecast methods often quite simple
 - Easy to explain
 - Easy to implement
 - Based on aggregated data - triangles
 - Fairly limited array of “standard” methods
- Usually “cookbook” approaches that seem to make sense
- Generally simple statements of what we think will happen in the future
- Only provide an “estimate” (whatever that means) with no sense as to likely variation or causes for the variation
- Not really amenable to “report cards” of performance



Example – Chain Ladder

- One of the oldest of the forecast methods for P&C actuaries
- Array cumulative data (claim counts, loss payments, case incurred losses, etc.) in a standard triangle

$$\begin{array}{cccc} P_{11} & P_{12} & \cdots & P_{1n-1} & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n-1} & \\ \vdots & \vdots & \ddots & & \\ P_{n-11} & P_{n-12} & & & \\ P_{n1} & & & & \end{array}$$

- Calculate link ratios (development factors) as ratios of amounts divide by next younger amounts
- Then pick “representative” ratios at each age



Example – Chain Ladder (Continued)

- Once you have selections calculate the cumulative product from more advanced ages to younger ages
- Multiply the cumulative amounts at each age by the product corresponding to that age
- These result in estimates for this particular method
- Note that they are simply that – point estimates
- They do not tell you anything more except that if
 - Amounts always move from age 1 to age 2 exactly as selected and
 - Amounts always move from age 2 to age 3 exactly as selected and
 - ...
 - Amounts always move from age n-1 to age n exactly as selected
- Then the ultimate amount will be thus and such



Example – Chain Ladder (Continued)

- Quick poll – show of hands – how many believe the ultimate amounts
 - Will actually exactly equal the chain ladder forecast?
 - Will usually come out close to the chain ladder forecast?
 - Will, on the average (after lots of trials) will equal the chain ladder forecast?
 - Will be above the chain ladder forecast as often as they are below?
- Now the more difficult question: Why?
- The second through fourth poll items ask questions that are only really answered in a statistical (stochastic) framework
- The Chain Ladder method as it is usually applied is not stochastic but rather deterministic



Traditional Reserving

- The more common traditional forecast (reserving) methods are:
 - Chain Ladder
 - Bornhuetter-Ferguson
 - Berquist-Sherman incremental severity method
- All are deterministic – say if amounts follow a certain specific pattern then their ultimate value will be thus and such
- Our forefathers long recognized that is unadvisable to place all your eggs in a single basket
- If there is a change in pattern then the forecasts will be affected
- The various methods have different characteristics and react to different influences in the data
- Thus traditionally multiple methods are considered



Traditional Reserving – Summary

- Traditional methods lay out a series of steps to follow
- Provide a deterministic forecast under the very restrictive assumption that each and every assumption is exactly correct
- Say nothing about
 - How likely the assumption is to be correct
 - How likely the assumption is “nearly” correct
 - Volatility of the forecasts
 - Whether emerging data is “reasonable” in light of prior selections
- Our forefathers overcame some of these limitations by using a number of methods and their experience with the methods to get their “best estimate”



SOME REPORTS FROM THE FRONTIER



December 10, 2009

First Some Basic Questions

Possible “estimates”

- The precise future outcome
- Something “close” to the future outcome
- The average possible future outcome
- The most likely future outcome
- An amount that is above the future outcome as often as it is below
- Something else

Statistical Concept

- Observation
- Probability interval
- Mean or expected value
- Mode
- Median
- ???



Simple Example

- Assume you are to set a reserve at year-end 2009 for a single claim to be settled on the next day at \$1,000 times the number of spots showing after the roll of a fair die
- From our prior slides the options we showed are:
 - Observation – got a crystal ball?
 - Middle third – between \$3,000 and \$4,000 inclusive
 - Mean – \$3,500
 - Mode – does not exist (all multiples of \$1,000 are equally likely)
 - Median – any number between \$3,000 and \$4,000 exclusive
- Notice here, unlike “traditional” reserving methods we know how likely any particular value is and if we had a number of such claims we can test whether they actually do follow this example



Some Attempts at Distributions of Outcomes

- Some early attempts in the 1980's and 1990's by the speaker
 - Forecasts of chain ladder if age-to-age factors assumed lognormal
 - Some collective risk applications attempting to recognize more than one are of uncertainty
- Mack in the late 1990's derived a non-parametric estimate of the standard error of chain ladder forecasts
- Verall and others in the late 1990's and early 2000's set the chain ladder in a stochastic framework and use bootstrap
- Stochastic differs from deterministic
 - Both are simplified (mathematical) models of “reality”
 - Stochastic adds explicit assumptions regarding the underlying uncertainty in a process



Chain Ladder in a Stochastic Framework

- Using prior notation for a triangle, the deterministic chain ladder method says that $P_{i2} = f_1 \times P_{i1}$ for each exposure year i
- Notice that there is no statement about how certain or uncertain the relationship is
- We can add a stochastic dimension to this example by adding a random component, resulting in $P_{i2} = f_1 \times P_{i1} + e_i$ where e_i is a random variable with mean 0
- Since e_i has a 0 mean, on the average the chain ladder relationship will hold but could be violated for individual observations
- This is the statement of a regression problem through the origin if each e_i is assumed to have the same normal distribution



Mack

- Notice the prior problem is only for factors at a single age and can be used to test whether or not the basic chain ladder assumptions actually hold
- Says nothing about the full chain ladder forecast for a single exposure year or all years combined
- For the full chain ladder the math gets messy quickly
- Mack made simplifying assumptions and gets an estimate of the standard error of the forecasts
- Says nothing about the distribution
- Many posit a distribution (e.g. lognormal) and use Mack estimates to derive the parameters of that distribution



Bootstrap

- Typically a non-parametric approach, that is makes no specific assumption regarding underlying distributions though there are parametric versions
- Basically assumes that the observed data gives the only information about the uncertainty
- In most simple version
 - Calculate expected development at an age using weighted averages or ratios of successive column sums
 - Calculate the variance in each column of the incremental forecasts
 - Calculate a triangle of standardized residuals
 - Square the triangle by generating forecasts perturbed by random draws (with replacement) from standardized residuals
- Repeat lots of times



Mack & Bootstrap

- Both are based solely on the chain ladder model
- Simply ways to estimate the volatility of chain ladder estimates
- Mack
 - Does not give the distribution, but only the standard error of forecasts
 - Some have questioned the validity of the various assumptions Mack makes
- For the bootstrap the only (standardized errors) that can occur are those that have occurred so presence or absence of a “rare” event can greatly skew results
- Each applied to a single data array (paid, incurred) and completely ignore model uncertainty



SAILING PAST THE EDGE OF THE WORLD



December 10, 2009

19

A Very Powerful Tool

- Looking at the regression problem touched on earlier
- We note that one way to estimate the parameter f_1 is to simply find the value of f_1 that minimizes the sum square errors of the forecasts given the observed data
- Notice no statistical statement here – pure calculus only
- But why is this a “good” way to estimate the parameter? Is there a “better” way?
- The answer is that in a statistical framework you can define what you mean by a “best” estimator, typically one that is least prone to error (i.e. lowest standard error)

▪ It happens that a particular estimator, the maximum likelihood estimator (MLE) fits this bill in a wide variety of situations



Maximum Likelihood Estimator

- Suppose we have a stochastic framework for a problem, with the distributions dependent on a (possibly vector valued) parameter
- For any given value of the parameter it is not difficult to calculate the likelihood of any given observation in our data
- The idea of the MLE is quite simple – for a given set of observations from a model, find the value of the parameter that maximizes the likelihood of that set of observations
- It turns out that not only is the MLE “best” in the sense above but it also has a number of very useful properties including an assessment of how uncertain the parameters themselves are



Sources of Uncertainty

- Process – random fluctuations in results even if all assumptions are known and correct – in our die example this is reflected in the fact that the result of a future roll is unknown
- Parameter – the chance that we have the model correctly specified but are not sure of the actual value of the parameters – neither Mack nor Bootstrap directly address this while MLE can provide valuable information here – in our die example this is reflected in the possibility that the die is not fair
- Model uncertainty – the chance that the model we have specified is wrong. None of the approaches outlined so far addresses this – as such each gives information about the volatility of the forecasts of a single method given observed data and not the distribution of outcomes for a reserve exercise



Use of MLEs

- MLEs have been in use for quite some time
- It turns out that the parameters that minimize the mean square error in the regression problem are MLEs if the errors are normal with constant variance
- Historically MLEs were derived analytically, often requiring quite a bit of analytic effort
- With the advent of reasonably powerful computers there is less need for analytic solutions with numerical solutions providing excellent approximations
- These numerical estimates can be used to estimate parameter uncertainty which can be used in simulations to derive a distribution of forecasts given a model and underlying data

Incremental Severity Model and Beyond

- Speaker has published a stochastic framework for the traditional Berquist-Sherman incremental severity model
- Provides estimates of both process and parameter uncertainty
- Can combine the two to approximate the distribution of forecasts from this model given the data using various approaches
 - Monte Carlo simulation
 - Gibbs Sampler
 - Metropolis-Hastings Algorithm
- The approach is quite flexible and can be adapted to a wide range of models



Addresses process and parameter, but not model uncertainty

Some Benefits of Stochastic Approach

- Information not only on expected forecast but also of potential volatility of forecasts
- Quantifiable answer to “How good is your forecast?”
- You can compare the actual residuals with expected residuals to see how well the stochastic assumptions fit with the data
- You can get constant monitoring of how well the emerging data fits with the model – you can see whether or not emerged data is reasonably expected given your model and if not you have early warning to change
- Bottom line – you get much more information than from traditional methods given the same underlying data



An Approach to Model Uncertainty

- So far, as with traditional methods we have not dealt with model uncertainty
- Probably the greatest source of uncertainty in actuarial projections on the P&C side
- Stochastic approaches can help in this regard as well
- You can
 - Apply a range of models
 - Calculate the likelihood of the data given the models
 - Use that likelihood to blend models by using Bayes Theorem
- If you had all the models in the world this would overcome model uncertainty, but using even a few models could result in better understanding and estimates

REDRAWING THE MAP



December 10, 2009

27

Summary

- Traditional methods have been useful and
 - Can, with appropriate skill and knowledge provide very useful information
 - Can give qualitative information about the uncertainty in projections
 - Do not provide any statistically meaningful information
- Popular stochastic approaches are a small step there, but are restricted to the forecasts of a single model and do not provide distribution of outcomes
- More robust methods are on the horizon that can help not provide more refined estimates than Mack and bootstrap but also overcome model uncertainty
- There is a brave new world beyond the edge of the earth!

